

# THE PSEUDO-EVOLUTION OF HALO MASS

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## ABSTRACT

A dark matter halo is commonly defined as a spherical overdensity of matter with respect to a reference density, such as the critical density or the mean matter density of the Universe. We show that such definitions lead to a spurious evolution in the halo's mass even if its physical density profile remains constant over time. This *pseudo-evolution* in mass is caused by the evolution of the reference density with redshift, and has little connection with the actual physical accretion of mass. We compute the pseudo-evolution of halos identified in a large N-body simulation from  $z = 1$  to 0, and show that it increases halo masses significantly across a wide range of halo masses and overdensities. Pseudo-evolution accounts for almost the entire mass evolution of halos with  $M_{200\bar{\rho}} \lesssim 10^{12} h^{-1} M_{\odot}$ , while for larger halos it still accounts for  $\sim 50\%$  of their overall mass evolution. We estimate the magnitude of the pseudo-evolution assuming that halo density profiles remain static in physical coordinates, and show that this simple model predicts the pseudo-evolution of simulated halos to a few percent accuracy. We discuss the impact of pseudo-evolution on the evolution of the halo mass function. We show that the non-evolution of the low-mass end of the halo mass function is the result of a fortuitous cancellation between pseudo-evolution and the absorption of small halos into larger hosts. We also show that the evolution of the low-mass end of the concentration-mass relation observed in simulations is almost entirely due to the pseudo-evolution of mass. Finally, we discuss the implications of our results for the interpretation of the evolution of various scaling relations between the observable properties of galaxies and galaxy clusters, and their halo masses.

*Subject headings:* cosmology: theory - methods: numerical - dark matter - galaxies: halos

## 1. INTRODUCTION

In a cold dark matter cosmological scenario (see e.g. Peebles 1982; Davis et al. 1985), the drama of galaxy formation unfolds at the virialized peaks of the density field, or *halos*. Although galaxies themselves are highly diverse, a number of their properties exhibit remarkable regularity and can be expressed as galaxy scaling relations. In particular, the stellar mass-halo mass relation and the luminosity-halo mass relation of central galaxies constrain important aspects of galaxy formation and have been studied via a variety of probes such as satellite kinematics (Prada et al. 2003; Conroy et al. 2007; More et al. 2009, 2011b), galaxy-galaxy weak lensing (Seljak 2000; McKay et al. 2001; Mandelbaum et al. 2006; Parker et al. 2007; Schulz et al. 2010), the abundance of galaxies and their clustering (Yang et al. 2003; Zehavi et al. 2004; Tinker et al. 2005; Zehavi et al. 2005; Skibba et al. 2006; van den Bosch et al. 2007; Brown et al. 2008; Conroy & Wechsler 2009; Moster et al. 2010, 2012; Behroozi et al. 2010; Yang et al. 2012a) or a combination of the above probes (Yoo et al. 2006; Cacciato et al. 2009; Leauthaud et al. 2012; More et al. 2012a). In order to understand the formation and evolution of galaxies, it is crucial to interpret the evolution of these scaling relations, which in turn requires a solid understanding of the evolution of halo masses with cosmic time.

Analogously, the largest halos in the Universe host clusters of galaxies, which themselves serve as laboratories for galaxy formation. The observable properties of clusters,

such as X-ray temperature, entropy profile, the mass of the intracluster-gas, or their evolution with redshift, are often described using a self-similar model (Kaiser 1986, see also Kravtsov & Borgani 2012 for a review). This model provides predictions for the scaling relations between halo mass and the observable properties of clusters. Large observational campaigns have been undertaken in the past (Vikhlinin et al. 2006; Böhringer et al. 2007; Mantz et al. 2010) and are also currently under way (e.g. Benson et al. 2011) to calibrate these scaling relations since they are necessary to obtain cosmological constraints from the observed abundance of clusters and its redshift evolution (e.g., Vikhlinin et al. 2009a, see Allen et al. 2011 for a recent review). However, such observational campaigns must be supplemented by sound theoretical models for the evolution of the scaling relations, which have still not been fully developed (see, e.g., a recent analysis by Lin et al. 2012).

When quoting the scaling relations between halo mass and galaxy (or galaxy cluster) properties, observers inevitably adopt a specific definition for the boundaries of halos, often based on the extent of their observations. However, numerical simulations show that dark matter halos exhibit smooth density profiles without well-defined boundaries, which makes the definition of the halo boundary and the associated halo mass ambiguous. The mass definition often used in the literature corresponds to the mass within a spherical boundary that encloses a given overdensity,  $\Delta(z)$ , with respect to a reference density,  $\rho_{\text{ref}}(z)$  (see, e.g., Cole & Lacey 1996). This spherical overdensity (SO) halo mass,  $M_{\Delta}(z)$ , and radius,  $R_{\Delta}(z)$ , are thus related via the following equation,

$$M_{\Delta}(z) = \frac{4}{3} \pi R_{\Delta}^3(z) \Delta(z) \rho_{\text{ref}}(z). \quad (1)$$

The most common choices of reference density are either the critical density,  $\rho_c$ , or the mean matter density,  $\bar{\rho}$ , of the uni-

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verse at a given cosmic epoch. The parameter  $\Delta$  can be chosen arbitrarily, but certain values such as  $\Delta = 180$  can be justified with the spherical top hat collapse model for an Einstein-de Sitter cosmology (Gunn & Gott 1972). The spherical collapse model has also been generalized to cosmological models which include a cosmological constant or non-zero curvature (Lahav et al. 1991; Lacey & Cole 1993; Eke et al. 1996).

The fundamental issue with the mass definition of Equation (1) is that the reference density evolves with cosmic time, leading to an evolution in halo mass even if the physical density profile of the halo is constant. For the remainder of this paper, we shall call the evolution of halo mass due to changing reference density *pseudo-evolution* because it is due solely to the mass definition and not to any actual physical mass evolution caused by the accretion of new material. Note that the actual evolution of spherical overdensity mass, which we shall call *mass evolution*, is a combination of the physical evolution due to the accretion of matter and pseudo-evolution (see Appendix A).

The fact that the evolution of the SO mass may not correspond to any actual physical evolution of mass has been pointed out before. Diemand et al. (2007) analyzed the accretion history of the Milky Way sized Via Lactea halo and found no significant physical growth after  $z = 1$ , even though the virial mass of the halo increased significantly. Prada et al. (2006) studied the outer regions of collapsed halos at  $z = 0$  and found no systematic infall for halos with masses lower than  $5 \times 10^{12} h^{-1} M_\odot$ . In a follow-up study, Cuesta et al. (2008) demonstrated a lack of physical accretion onto galaxy mass halos, and proposed an alternative mass definition that aims to include all mass bound to a halo (see also Anderhalden & Diemand 2011). Although such a mass definition may be more physical and closer to the meaning of mass in analytical models of halo collapse and evolution, its observational analogue is very difficult or even impossible to measure for real systems. Thus, the SO mass is most often used in observations, and a proper interpretation of observational results should take into account the pseudo-evolution inherent in this mass definition. For the case of cluster scaling relations, Kravtsov & Borgani (2012) argued that part of their evolution is due to pseudo-evolution.

In this paper we seek to quantify the pseudo-evolution of the SO halo mass due to an evolving reference density. In particular, we focus on quantifying the contribution of the pseudo-evolution to the mass accretion history (MAH) of halos, since the MAH describes the dark matter part of the observable mass scaling relations for galaxies and galaxy clusters. Much work has been invested into quantifying MAHs, but the contributions from physical accretion and pseudo-evolution are generally not distinguished (see e.g. Wechsler et al. 2002; van den Bosch 2002; Zhao et al. 2003; Miller et al. 2006; Zhao et al. 2009). We will also investigate the impact of pseudo-evolution on the evolution of the halo mass function.

This paper is structured as follows. In §2, we describe a simple analytical model based on the NFW density profiles (Navarro et al. 1997) to gauge the contribution of pseudo-evolution to the total MAH. In §3, we quantify the pseudo-evolution of actual halos in cosmological simulations and show that the results are in good agreement with the analytic model of §2. We discuss caveats and implications of our results as well as directions for future work in §4, and give a summary of our results in §5. Throughout this paper, we denote overdensities as  $\Delta_c$  if they are defined relative to  $\rho_c$ , and

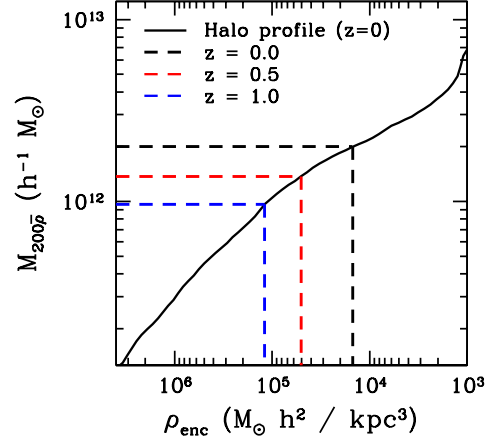


FIG. 1.— Visualization of the static halo model. The solid line shows the spherical mass profile as a function of enclosed density ( $M/V$ ) for a halo of mass  $M_{200\bar{\rho}} = 2 \times 10^{12} h^{-1} M_\odot$  from the Bolshoi simulation at  $z = 0$ . The x-axis is reversed, so that the left side of the plot corresponds to the high-density center of the halo, and the right to the low-density outskirts. The vertical dashed lines indicate the spherical overdensity  $200\bar{\rho}$  at redshifts 0, 0.5 and 1, and the horizontal dashed lines mark the corresponding halo mass  $M_{200\bar{\rho}}$ . Since the reference density  $\bar{\rho} \propto (1+z)^3$ , the halo density threshold increases with redshift, and  $M_{200\bar{\rho}}$  decreases. Even if the physical mass distribution of this halo was kept fixed between  $z = 1$  and  $z = 0$ , its mass  $M_{200\bar{\rho}}$  would undergo a *pseudo-evolution* from  $9.6 \times 10^{11} h^{-1} M_\odot$  to  $2 \times 10^{12} h^{-1} M_\odot$ .

$\Delta_m$  if they are defined relative to  $\bar{\rho}$ . We also use  $\Delta_{\text{vir}}$  to denote the redshift and cosmology-dependent virial overdensity predicted by the spherical collapse model, which corresponds to  $\Delta_{\text{vir}} \approx 358$  at  $z = 0$  and  $\Delta_{\text{vir}} \approx 180$  at  $z > 2$  with respect to the mean background density for the concordance fiducial cosmology used in this paper (e.g., Bryan & Norman 1998). All densities and radii are expressed in physical units, unless stated otherwise.

## 2. THE STATIC HALO MODEL

Let us consider a density peak in the Universe around which the matter density profile in physical units has not evolved since a given redshift,  $z_0$ . As shown in Fig. 1, the halo mass associated with this density peak will change purely due to the evolution of the reference density used to define its boundary. This evolution in mass can be quantified using the density profile of the halo at redshift  $z_0$ . Let us assume that the density distribution around this density peak is described by the universal density profile given by

$$\rho(r, z_0) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}, \quad (2)$$

which has been found to be a reasonable approximation of the typical density profiles around density peaks in cold dark matter cosmologies (Navarro et al. 1997, NFW hereafter). The scale radius  $r_s$  and the halo radius  $R_\Delta$  are related by the concentration parameter  $c_\Delta = R_\Delta/r_s$ . Under our assumption that the density profile around this peak does not evolve and that profile of Equation (2) is a good description of the actual profile at the radii of interest, the halo mass at any redshift  $z$  can be expressed in terms of the characteristic density  $\rho_s$  and  $r_s$ , by integrating this *static* density profile within the halo radius  $R_\Delta(z)$ , such that

$$M_\Delta(z) = \int_0^{R_\Delta(z)} \rho(r) 4\pi r^2 dr = \rho_s 4\pi r_s^3 \mu[c_\Delta(z)], \quad (3)$$

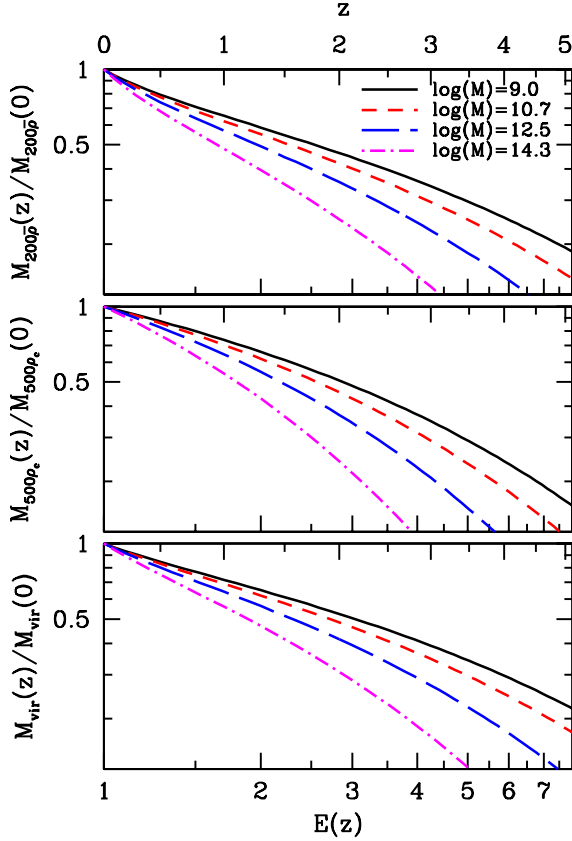


FIG. 2.— Predictions of the static halo model. Lines show the *pseudo-evolution* of halo mass due to changing reference density as a function of  $E(z)$  relative to the halo mass at  $z = 0$ . The labels indicate the halo mass in  $\log(h^{-1}M_\odot)$ . From top to bottom, the panels show the evolution of  $M_{200^c}$ ,  $M_{500^c}$  and  $M_{\text{vir}}$ , where  $\bar{\rho}$  and  $c$  indicate the average and critical densities of the universe, respectively, and  $M_{\text{vir}}$  corresponds to an evolving overdensity according to Bryan & Norman (1998). For  $z < 0.5$ , the pseudo-evolution is largest for  $M_{200^c}$ , but the overall trend is the same for all three mass definitions.

where the function  $\mu[x]$  is given by

$$\mu[x] = \ln(1+x) - \frac{x}{1+x}. \quad (4)$$

Equating the right hand sides of Equations (1) and (3), we obtain a relation between the concentration parameter of the halo at redshift  $z$  and the concentration parameter at redshift  $z_0$ ,

$$\frac{c_\Delta(z)^3}{\mu[c_\Delta(z)]} = \frac{3\rho_s}{\Delta(z)\rho_{\text{ref}}(z)} \quad (5)$$

$$= \frac{c_\Delta(z_0)^3}{\mu[c_\Delta(z_0)]} \left[ \frac{\Delta(z_0)\rho_{\text{ref}}(z_0)}{\Delta(z)\rho_{\text{ref}}(z)} \right]. \quad (6)$$

This relation can in turn be used to find the evolution of halo mass according to the equation

$$M_\Delta(z) = M_\Delta(z_0) \frac{\mu[c_\Delta(z)]}{\mu[c_\Delta(z_0)]}. \quad (7)$$

As examples, we consider three commonly used definitions of halo mass in the literature, (i)  $\Delta_m(z) = 200$  as in studies of the halo occupation distribution of galaxies, (ii)  $\Delta_c(z) = 500$  as in studies involving galaxy cluster observations, and (iii)  $\Delta_c(z) = \Delta_{\text{vir}}$ . We assume that the density profile around a density peak does not change from redshift  $z = 1$  to  $z = 0$ . With-

out loss of generality, we use  $z_0 = 0$  to define the static density profile in physical units. We consider the concentration-mass relation at  $z = 0$  given by Zhao et al. (2009, hereafter Z09), and use Equations (6) and (7) to obtain the mass evolution due to pseudo-evolution.

Fig. 2 shows the mass evolution histories for halos of different masses as predicted by our static halo model. The different panels correspond to the three commonly used overdensity definitions. Each panel shows the pseudo-evolution of mass from  $z = 0$  to  $z = 5$ , normalized to the halo's mass at  $z = 0$ , as a function of the expansion rate in units of the Hubble constant,  $E(z)$ . Assuming a flat  $\Lambda$ CDM cosmology,  $E(z)$  is defined as usual,

$$E(z) = \sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}. \quad (8)$$

As expected, more massive halos undergo a larger evolution due to the lower values of their concentrations. Regardless of the exact mass definition, the fractional change in halo mass due to pseudo-evolution can be as large as  $\sim 0.5$  by  $z = 1$ . The shape of the mass evolution history with redshift is not only a function of halo mass, but also depends on the exact mass definition. Its functional form is better approximated by a power law of  $E(z)$  than  $(1+z)$ , but still shows deviations from an exact power law behaviour.

### 3. HALO MASS EVOLUTION IN SIMULATIONS

#### 3.1. Numerical simulation

To quantify the pseudo-evolution of mass using realistic halo profiles, we use a sample of halos extracted from a dissipationless cosmological simulation of the  $\Lambda$ CDM model. Specifically, we use the Bolshoi simulation (Klypin et al. 2011), which followed the evolution of the matter distribution using the Adaptive Refinement Tree (ART) code (Kravtsov et al. 1997; Gottloeber & Klypin 2008) in a flat  $\Lambda$ CDM model with parameters  $\Omega_m = 1 - \Omega_\Lambda = 0.27$ ,  $\Omega_b = 0.0469$ ,  $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) = 0.7$ ,  $\sigma_8 = 0.82$  and  $n_s = 0.95$ . These cosmological parameters are compatible with measurements from WMAP7 (Jarosik et al. 2011), a combination of WMAP5, BAO and SNe (Dunkley et al. 2009), X-Ray cluster studies (Mantz et al. 2008; Vikhlinin et al. 2009b), and observations of the clustering of galaxies and galaxy-galaxy/cluster weak lensing (see e.g. Tinker et al. 2012; More et al. 2012b). The same cosmology was used for the calculations shown in Fig. 2, and will be used for the remainder of this paper.

The Bolshoi simulation uses  $2048^3 \approx 8$  billion particles to follow the evolution of the matter distribution in a cubic box of size  $250 h^{-1} \text{ Mpc}$  on each side, which corresponds to a particle mass of  $1.35 \times 10^8 h^{-1} M_\odot$ . This implies that the smallest halos considered in this paper ( $M_{\text{vir}} = 2 \times 10^{11} h^{-1} M_\odot$ ) are resolved by over 1000 particles. As density peaks, we use the centers of halos from a catalog generated with the bound density maxima (BDM) algorithm (Klypin & Holtzman 1997; Klypin et al. 2011).

We identified all distinct halos with  $M_{\text{vir}} \geq 2 \times 10^{11} M_\odot$  from the simulation at  $z = 0$ , resulting in a sample of about 240,000 halos. For each of these halos, we constructed radial density profiles by summing the particle contributions in 80 logarithmically spaced bins, spanning radii from  $0.05$  to  $10 R_{\text{vir}}$ . As a sanity check, we extracted density profiles for the same Bolshoi halos from the MultiDARK database (Riebe et al. 2011), compared them to ours and found excellent agreement. The larger radial range and finer resolution of our profiles com-



pared to the ones existing on the database allowed us to define halo masses for lower overdensity thresholds of  $\Delta_m = 200$ , and much lower background densities than those used to define the catalog.

For the purposes of this paper, we focus on the mass evolution from  $z = 1$  to  $z = 0$ . Before  $z \sim 1$ ,  $\Omega_m \sim 1$ , meaning that physical accretion dominates for halos in the mass range of interest (Z09). Thus, the impact of pseudo-evolution is expected to be largest from  $z = 1$  to  $z = 0$ . Furthermore, most observational determinations of galaxy and galaxy cluster scaling relations have been performed below  $z = 1$ , which makes this particular redshift range the most interesting to investigate.

### 3.2. The evolution of $M_{200\bar{\rho}}$ and $M_{500\rho_c}$

In a first experiment, we focus on mass definitions based on the mean matter density of the universe, such as  $M_{200\bar{\rho}}$ . To estimate the pseudo-evolution of halo mass using realistic matter density profiles, we assume that the density profiles stay constant in physical units from  $z = 0$  to  $z = 1$  and evolve the background density according to

$$\bar{\rho}(z) = (1 + z)^3 \bar{\rho}(0). \quad (9)$$

The radius,  $R_{200\bar{\rho}}$ , of the halo is then numerically identified to be the radius which encloses an average overdensity of  $\Delta = 200$  with respect to  $\bar{\rho}(z)$ , and the mass,  $M_{200\bar{\rho}}$ , follows from Equation (1).

In the left hand panel of Fig. 3, we show the mean of the ratio of the evolved halo mass to the halo mass at redshift zero as a function of redshift using open circles, and the 16 and 84 percentiles of the distribution of this ratio as the error bars. The analytical estimate from §2 is shown using a solid line, with gray contours indicating the 68% confidence interval. This uncertainty in the analytical prediction is due to scatter in the concentration-mass relation, which we assumed to be 0.14 dex based on the results of Wechsler et al. (2002). The excellent agreement between the analytical estimate and the results from the halo profiles implies that our assumption about the density distribution at redshift zero (from the models of Z09) is not too far off from the actual density distribution of halos in the simulation. This shows that the analytical model can provide an excellent description of the mass evolution if the physical density distribution of the halos was indeed constant.

Next, we would like to contrast the predictions of the static halo model with the *true* mass evolution histories of halos observed in simulations. In principle, we could derive the true mass evolution history of a halo by constructing merger trees from the simulation, and following the main progenitor branch of the halo. Instead, for simplicity, we make use of the Z09 model for the mass evolution histories of halos. This model has been shown to accurately reproduce the mass evolution histories for a large variety of cosmological models (scale-free or  $\Lambda$ CDM). The results from this model will thus include the effects of both pseudo-evolution and the actual physical accretion of mass. By comparing the mass evolution histories from our static halo model to these realistic mass evolution histories, we can disentangle the two effects.

The mass evolution histories predicted by the model of Z09 are shown by dashed lines in Fig. 3. The comparison clearly shows that a significant fraction of the halo mass evolution is due to the evolving reference density in the mass definition. In fact, for halos of mass  $M_{200\bar{\rho}} \lesssim 10^{12} h^{-1} M_\odot$ , the mass evolution history at  $z \lesssim 1$  can be explained almost entirely by

the static halo model, and true physical accretion constitutes a negligible portion of the overall evolution. For high-mass halos ( $M_{200\bar{\rho}} \sim 10^{14} h^{-1} M_\odot$ ), the pseudo-evolution accounts for roughly 60 percent of the mass evolution.

There is, however, one potential issue with these results. By defining the static density profiles at redshift zero and working *backwards* in time, we always shift the boundary of halos radially inwards to define halos at higher redshifts. This means that our experiment does not exclude the possibility that the density profile in the outer parts of the halo (beyond the boundary) change at higher redshifts. This could very well be the case, if halos underwent an inside-out growth by primarily accreting mass at the outskirts as time progresses. However, this issue can be easily addressed by defining the static density profiles at redshift  $z_0 = 1$  and working *forward* in time. This allows us to test whether the density profiles of the halos follow the NFW form given by Equation (2) even in the outskirts of halos.

The open circles in the right hand panels of Fig. 3 show the result of using the density profiles from the simulation at redshift  $z_0 = 1$  and determining how the mass should grow as redshift decreases. We call this experiment a test of the *forward evolution* of the halo masses. Note that the forward evolution panels show the inverse of the evolved mass in units of the mass at  $z_0$ , such that equal numbers in the left and right hand panels correspond to equal amounts of halo growth. As in the left hand panel, we show the prediction from our static halo model as a solid line. If the physical density profile in the outskirts of halos at  $z = 1$  was different from what it is at  $z = 0$ , we would see a difference between the prediction of our analytical model (which assumes that the profiles are NFW all throughout) and the results from the actual density profiles at  $z_0 = 1$  from the simulation. However, the excellent agreement between the two results shows that the density profiles indeed are described, to a good approximation, by the NFW form even in the outskirts of halos that we considered at  $z_0 = 1$ . In Fig. 3 we also show the prediction of the mass evolution history from the models of Z09 using dashed lines. The fair agreement between our predictions of the static halo model with the mass evolution histories from Z09 at low halo masses ( $M_{200\bar{\rho}} \lesssim 10^{12} h^{-1} M_\odot$ ) validate our basic premise that the density profiles of such halos are already established at  $z = 1$  and do not change with time.

The mass evolution shown in Fig. 3 refers to a sample of isolated halos only. Inevitably, this sample contains close pairs of halos which are not identified as overlapping in the halo catalog, but whose density profiles pick up contributions from particles belonging to the other halo. As the density profiles we use extend as far as  $10 R_{\text{vir}}$ , this is the case for a significant number of halos in our sample. When we evolve such a halo forward in time, its radius grows, and the contribution from other halos leads to excess mass growth, which manifests itself as a large scatter in the mass evolution history. However, many of these halos end up lying inside the radius of a neighbouring, larger halo, and we need to exclude them from the averaged mass evolutions shown in Fig. 3.

In principle, the most straightforward way to identify sub-halos would be to use merger trees of the Bolshoi simulation. However, merger trees are generated with knowledge of the full mass evolution of halos (rather than just the pseudo-evolution), as well as their motion and acceleration. In the spirit of our extremely simple model of static halo profiles, we wish to avoid using such information, and rely only on the density profiles and initial positions of the halos in our sam-

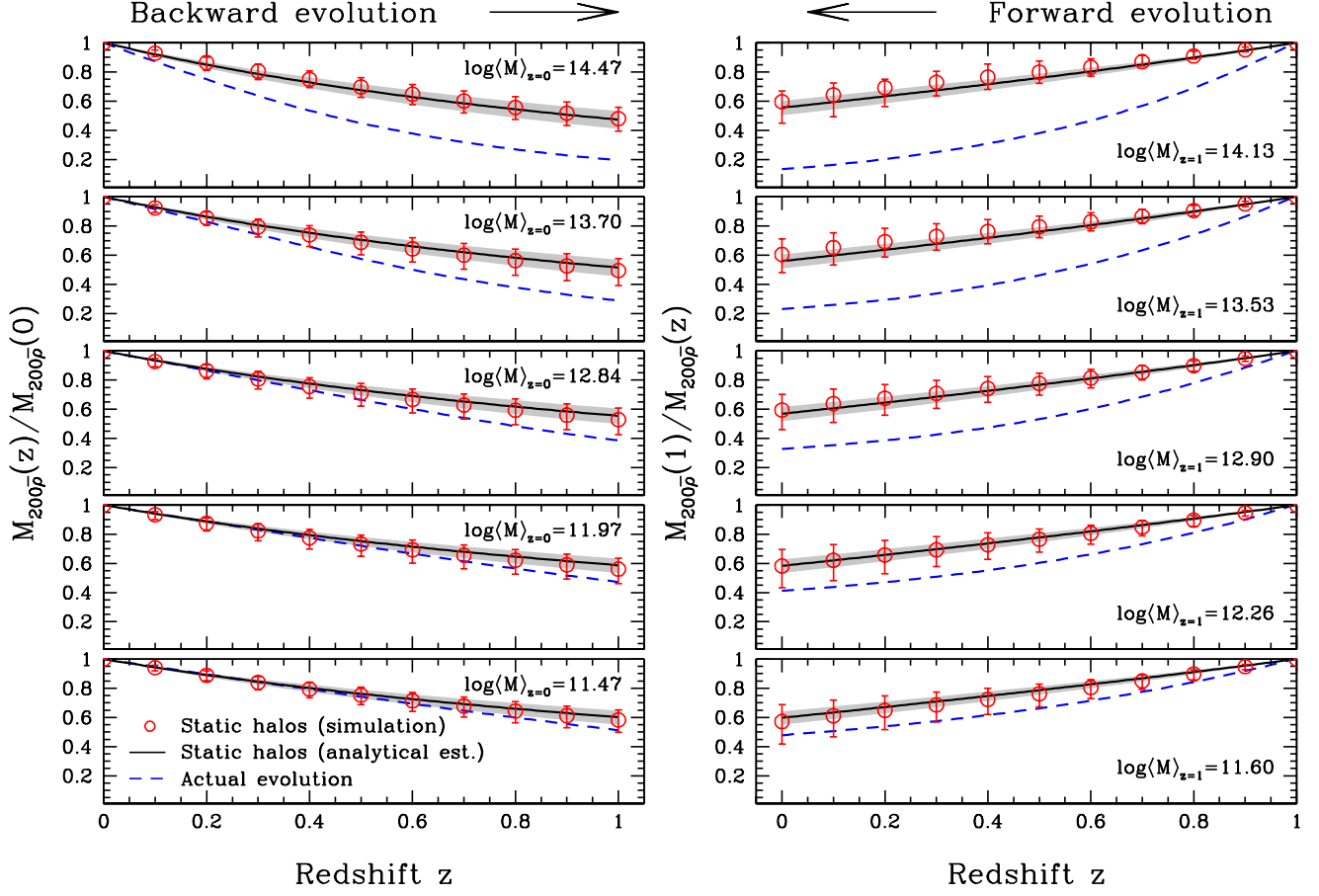


FIG. 3.— Mass evolution for different halo masses for the  $M_{200\bar{\rho}}$  mass definition, in five logarithmic mass bins. The dashed lines show the actual mass evolution of halos as predicted by the Z09 model. The solid lines show the evolution of mass due solely to the evolution of the reference density in the mass definition (*pseudo-evolution*) as predicted by the static halo evolution model (see §2). The gray band around the solid lines shows the 68% confidence interval due to scatter in the concentration-mass relation. The red points show the pseudo-evolution computed using density profiles from the Bolshoi simulation, with error bars indicating the mass range containing 68% of the halos in a given mass bin. The left panels show this evolution computed by extrapolating the mass evolution using profiles at  $z_0 = 0$  and going backwards in time, while the right panels show the evolution of profiles at  $z_0 = 1$ , going forward in time. Note that the right hand panels show the inverse of  $M(z)$ , meaning that equal values at  $z = 1$  in the left hand panels and  $z = 0$  in the right hand panels indicate the same amount of evolution between  $z = 1$  and 0. However, the mass bins on the left and right do not correspond to the same halo masses, and should thus not be compared directly. These results demonstrate that the pseudo-evolution accounts for about a factor of two at all halo masses. For small halos ( $M_{200\bar{\rho}} \lesssim 10^{12} h^{-1} M_\odot$ ), this represents the majority of their overall evolution, while for very large halos it accounts for about half. See §3.2 for further discussion.

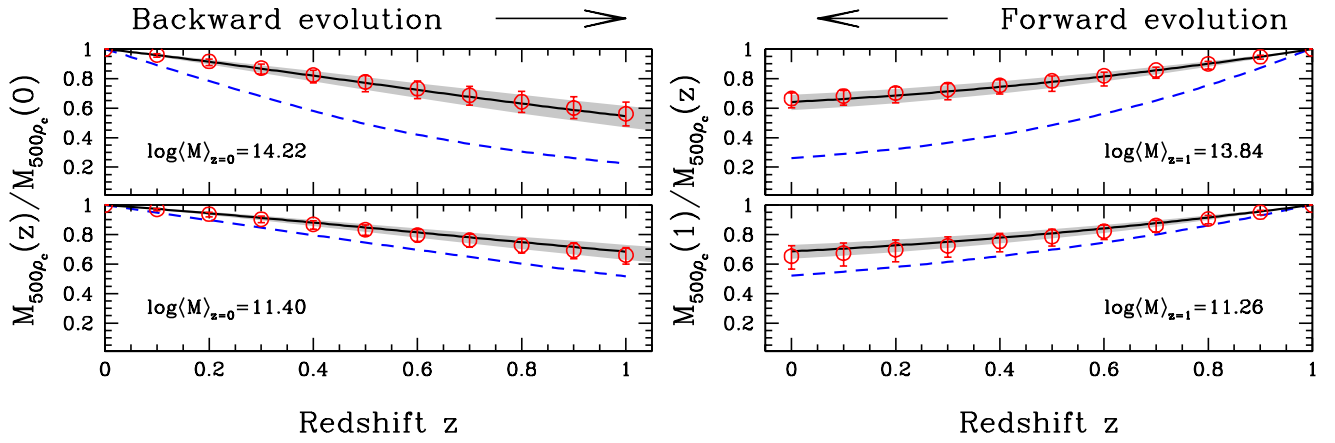


FIG. 4.— Like Fig. 3, but for the  $M_{500\rho_c}$  mass definition. Only the highest and lowest of five mass bins are shown. The analytical prediction of the static halo model and the results from simulated halos match even better than for  $M_{200\bar{\rho}}$ , and the Bolshoi results exhibit smaller scatter. This is caused by a smaller virial radius  $R_{500\rho_c}$ , meaning that irregularities in the outskirts of halos play a lesser role. The pseudo-evolution is a little weaker in  $M_{500\rho_c}$  than in  $M_{200\bar{\rho}}$ , but still accounts for almost all of the mass evolution at low halo masses.

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In the case of *backward evolution*, halo radii decrease, meaning that we do not need to worry about halos becoming subhalos. In the case of *forward evolution*, we identify subhalos as follows. At each redshift, we evolve the virial radii of all halos to match the evolved reference density. We then check whether the new virial radius of such a halo encloses the center of any other, smaller halos. Note that we assume that the neighboring halos stay at a constant physical distance and are not part of the Hubble flow, consistent with our assumption that the physical density around the peak does not evolve. We exclude the subhalos discovered in this manner from the current redshift bin, and all subsequent smaller redshifts. We start this process with the largest halo in the sample, marching down to the smallest halos. Once a halo has been found to be a subhalo, it cannot itself be the host of another halo. By the end of the evolution from  $z_0 = 1$  to  $z = 0$ , a total of about 14% of the halos in the sample had become subhalos and been removed from the sample. The mass evolution averages shown in Fig. 3 are insensitive to the removal of subhalos, but the scatter is reduced significantly by this procedure.

Following our discussion of mass definitions based on the mean matter density of the universe such as  $M_{200\bar{\rho}}$ , we now investigate definitions based on the critical density. Note that the difference between those definitions is not only due to the different values for  $\Delta$  which are typically chosen, but that  $\rho_c$  evolves qualitatively different from  $\bar{\rho}$  such that

$$\rho_c(z) = \bar{\rho}(z) \left[ \frac{E^2(z)}{\Omega_{m,0}(1+z)^3} \right]. \quad (10)$$

Fig. 4 shows the evolution histories of  $M_{500\rho_c}$  for two mass bins, compared to the true evolution represented by the model presented by Z09. The results were derived in exactly the same way as the results for  $M_{200\bar{\rho}}$ , except for the different evolution of  $\Delta\rho_{\text{ref}}$ . The pseudo-evolution is slightly weaker in  $M_{500\rho_c}$  than  $M_{200\bar{\rho}}$ . This can be seen by comparing, for example, the lowest mass bins in Figs. 3 and 4. The weaker evolution in  $M_{500\rho_c}$  may seem slightly counter-intuitive at first, since  $R_{500\rho_c}$  is *smaller* than  $R_{200\bar{\rho}}$  (which implies that  $c_{500\rho_c} < c_{200\bar{\rho}}$ ), and mass profiles as a function of enclosed density tend to steepen towards the center of halos (see Fig. 1). However, the weaker evolution of  $\rho_c$  compared to  $\bar{\rho}$  more than offsets this effect. For example, at  $z = 1$  the mean matter density of the universe was a factor of 8 higher than today, but the critical density was only larger by a factor of  $E^2(1) \approx 2.9$ .

### 3.3. The mass (non-) evolution of low-mass halos

One of the most striking consequences of the results presented in Figs. 3 and 4 is that the physical density profiles of low-mass halos ( $M_{200\bar{\rho}} \lesssim 10^{12} h^{-1} M_\odot$ ) barely change after  $z = 1$ , implying that they undergo negligible physical accretion. We seek to demonstrate this directly in Fig. 5 which shows the average radial velocities of particles around their halo centers at  $z = 1$ . The colored lines correspond to three narrow mass bins. We extracted the velocity information from the Bolshoi simulation using the same binning scheme as for the density profiles. As Cuesta et al. (2008) pointed out, the average radial velocities of low-mass halos amount to only a small fraction of  $v_{200}$  (the halo circular velocity at  $R_{200\bar{\rho}}$ ), while they grow to about half of  $v_{200}$  for the most massive bin in Fig. 5.

We estimated the average mass infall rates at every radius using the density profile and the average infall velocity profile,

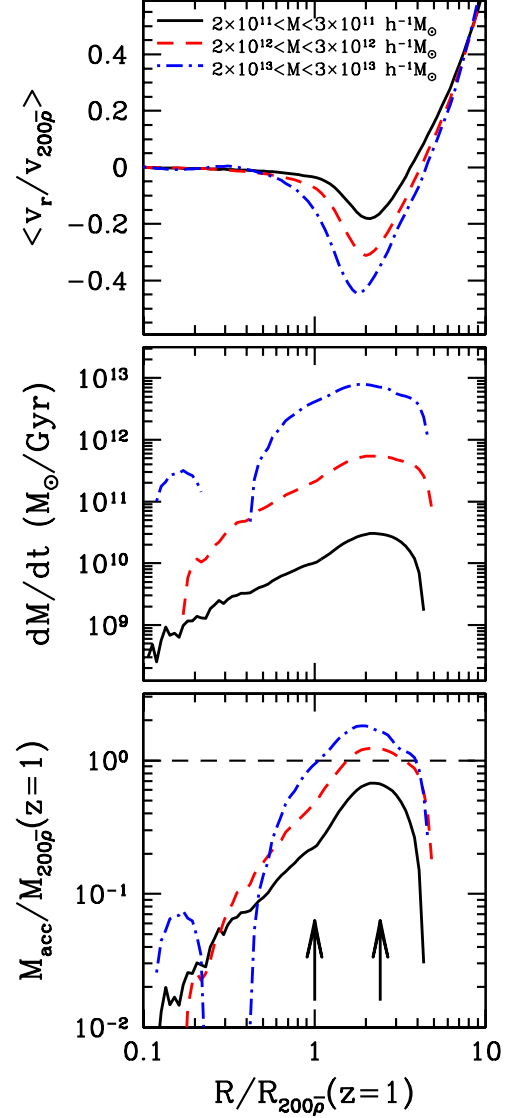


FIG. 5.— Mass infall around halos at  $z = 1$ . Top panel: Average radial velocity profiles of halos in three narrow mass bins, in units of the circular velocity  $v_{200} = \sqrt{GM_{200}/R_{200}}$  (compare to Fig. 13 in Cuesta et al. 2008). For low-mass halos, the average infall velocity never exceeds a small fraction of their circular velocity. Beyond a few virial radii, the Hubble flow dominates and drags matter away from the halo. Center panel: average mass flux as derived from the velocities in the top panel, and density profiles. The out-flowing parts of the profiles (positive radial velocity) are omitted. Bottom panel: an estimate of the mass accreted since  $z = 1$ . We assume that the mass flux has remained constant since  $z = 1$  (or 7.84 Gyr), and compute the total mass passing through a radial shell as a fraction of the original halo mass. The two arrows in the bottom panel indicate  $R_{200\bar{\rho}}$  at  $z = 0$  (right arrow) and  $z = 1$  (left arrow). See §3.3 for a discussion of these results.

$v_r(r)$ , using

$$\frac{dM}{dt} = 4\pi r^2 \rho(r) v_r(r). \quad (11)$$

These mass infall rates are shown in the center panel of Fig. 5 for halos corresponding to the three mass bins. We estimate the total accreted mass since  $z = 1$  by assuming that the radial velocity profiles stay constant, the same way the density profiles remain fixed. Given this fixed inflow profile, we extrapolate the total mass passed through a radial shell by multiplying the mass infall rate by the time since  $z = 1$  (bottom

panel of Fig. 5).

We can now compare this estimate of the physically accreted mass to the mass evolution shown in Fig. 3. According to Appendix A, we need to consider the physical mass added within the radius the halo reaches at the end of its pseudo-evolution (see the second integral in Equation A2). For the case of *backward evolution*, this radius is  $R_{200\bar{\rho}}(z = 1)$  (marked with the left arrow in the bottom panel of Fig. 5), and we consider the infall rates at that radius. Note that the accreted mass is shown in units of the halo mass at  $z = 1$ . Since halos in the lowest mass bin,  $M_{200\bar{\rho}} \approx 2 \times 10^{11} h^{-1} M_{\odot}$ , grow by about a factor of 2 by  $z = 0$  (mostly through pseudo-evolution), the infall rate at  $R_{200\bar{\rho}}(z = 1)$  corresponds about 10% of the halo mass today. This is in good agreement with the mass evolution results in Fig. 3. In the case of *forward evolution*, we need to consider the mass accreted within  $R_{200\bar{\rho}}(z = 0)$  (indicated by the right arrow in the bottom panel of Fig. 5). It appears that the accreted mass at that radius slightly overestimates the actual mass evolution for low-mass halos, and slightly underestimates the mass evolution of high-mass halos, which is presumably due to our assumption of fixed mass infall profiles. We intend to investigate this assumption in future work.

### 3.4. Halo mass function from the static halo model

Given the success of the static halo model in reproducing mass evolution histories for small mass halos, we would like to investigate the impact of the mass definition on the evolution of the halo mass function. For this purpose, we start from the  $z_0 = 0$  halo mass function calibration of Tinker et al. (2008, hereafter T08), and use the mass evolution history (backward evolution) inferred from our static halo model to predict the resulting evolution of the halo mass function. In the static halo model, the mass assigned to density peaks becomes smaller as redshift increases, which results in a mass-dependent shift of the halo mass function ( $MdN/dM$ ) towards the left. In order to quantify this effect, we used the same procedure as for the halo mass evolution. We selected the density profiles of halos of mass  $M_{\text{vir}} > 2 \times 10^{11} h^{-1} M_{\odot}$  extracted from the simulation and calculated the expected evolution of mass assuming that the density profiles around peaks stay constant in physical units.

Before comparing these results to the *actual* physical evolution of the mass function observed in numerical simulations, we first establish that these results match the analytical prediction of Equation (7). Since the corresponding mean mass evolution histories agree to a few percent (Fig. 3), we naively expect good agreement between the mass functions as well. However, Fig. 3 also reveals significant scatter in the pseudo-evolution of simulated halos, which could cause disagreement with the analytically predicted mass function. The comparison between the two estimates is shown in Fig. 6. The analytical prediction of the static halo model for  $z = 0.5$  and  $z = 1$  is shown with solid lines, while the actual halo mass function inferred from the pseudo-evolution of profiles in the simulation is shown by points. The fractional residuals between the evolution from the actual density profiles and the prediction from the static halo model are shown in the bottom panel. At both redshifts, the prediction from our static halo model agrees extremely well (to better than 5 – 10%) with the expected evolution from the actual profiles of the halos at  $z = 0$ . For the case of mass definitions using higher overdensities, such as  $M_{500\rho_c}$ , we expect even better agreement, since the static halo describes the pseudo-evolution of halos

in simulations more accurately (see Fig. 4).

The right hand panel of Fig. 6 shows the same comparison as the left hand panel, but for the case of *forward evolution*. For the prediction of the static halo model, we start from the halo mass function of T08 at  $z_0 = 1$ , and evolve forward in time to  $z = 0.5$  and  $z = 0$ . The slightly larger discrepancy at low masses at  $z = 0$  ( $\sim 10\%$ ) is an artefact due to the mass cut of  $M > 2 \times 10^{11} h^{-1} M_{\odot}$  that we imposed at  $z_0 = 1$ . Some halos which were excluded due to their low mass would have developed masses included in the shown range, leaving the low-mass end depleted.

Note that, unrelated to the static halo model, there is a discrepancy of  $\sim 5 - 10\%$  between the  $z_0 = 0$  and  $z_0 = 1$  calibration of the T08 mass function and the mass function obtained from the Bolshoi simulation. Given this initial offset at  $z_0$ , we cannot expect a smaller discrepancy at subsequent redshifts.

Furthermore, we considered the impact of statistical bias due to the presence of scatter in the mass evolution histories (Eddington 1913). It is evident from Fig. 3 that there is a non-negligible scatter in the mass evolution histories, and that this scatter is somewhat larger in the forward evolution case than in the backward evolution case. As the number density of halos is a decreasing function of halo mass, the number of halos that get up-scattered into a particular mass bin can be larger than the number of halos that get down-scattered out of that mass bin. However, we found that this bias does not influence the results appreciably.

### 3.5. Comparison with the true mass function

Having convinced ourselves that the halo mass functions predicted by the static halo model and halo profiles are consistent, we now wish to investigate the impact of pseudo-evolution on the *true* evolution of the halo mass function, analogous to the comparison of our mass evolution history with that predicted by the models of Z09. We use the calibration of the mass function provided by T08 to reflect the true evolution of the mass function measured in simulations. In Fig. 7, we present the comparison of the T08 mass function at three different redshifts with the evolution of the mass function due to the pseudo-evolution of density profiles from the simulation. Note that the residual panel includes a wider range than in Fig. 6. Let us first focus on the left hand panel which shows the *backward evolution* case. Since our estimates of the mass evolution histories of low mass halos matched those observed in simulations (Fig. 3), we expect good agreement with the mass function at the low-mass end, and discrepancies predominantly at the high-mass end. The left hand panel of Fig. 7, however, reveals significant disagreement at both mass ends. Since the cause of deviations are different at the low and high-mass ends, we shall discuss those regions separately.

At the high-mass end, we have shown that the growth of high-mass halos is largely due to physical accretion rather than pseudo-evolution. This rapid growth implies that the T08 mass function decreases strongly with redshift. As we evolve backwards in time, the progenitors of high-mass halos would need to be more massive than in reality if the growth was solely due to changing mass definition. The static halo model, therefore, overpredicts the number of large halos at  $z = 1$ .

At the low-mass end, computing only the pseudo-evolution underpredicts the T08 calibration at  $z = 0.5$  and 1 by roughly 20 – 30% (left panel of Fig. 7). From the calibration of T08, it appears that the number density of low mass halos



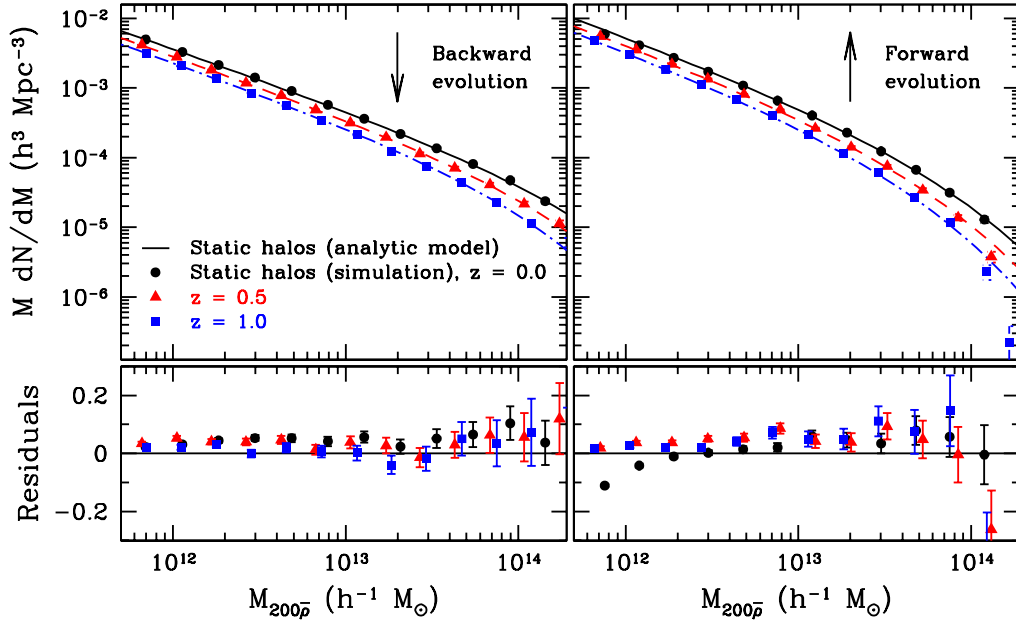


Fig. 6.— Static halo model predictions for the halo mass function in three redshift bins. The data points show the pseudo-evolution as derived from the density profiles of Bolshoi halos, evolving backward in time (left panel) and forward in time (right panel). The error bars indicate the Poisson uncertainty due to the limited number of objects in the Bolshoi sample. The solid lines show the analytical estimate of Equation (7), with residuals plotted in the bottom panels. The analytical estimate and results from simulated profiles agree very well at all redshifts. Since the analytical estimate is based on the T08 mass function at  $z_0$ , some scatter is inevitable due to calibration uncertainties in the T08 mass function itself. At the low-mass end of the forward evolution (right panel), the simulated sample seems to underestimate the mass function. This is simply due to the mass cut made in the simulated sample, and the resulting lack of low-mass halos when evolved to  $z = 0$ . See §3.4 for a detailed discussion of these results.

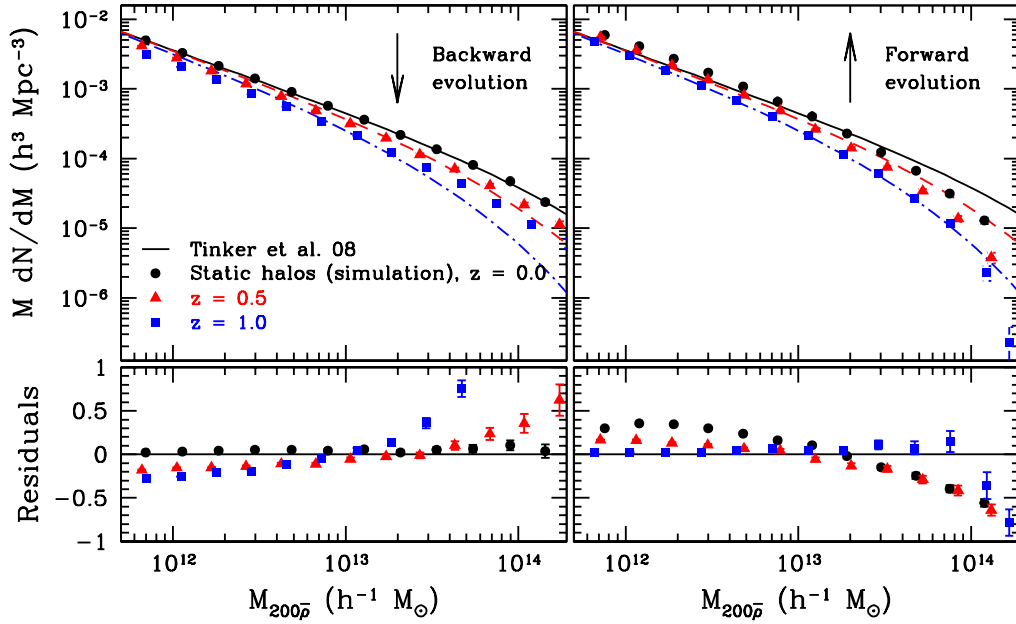


Fig. 7.— Comparisons of the mass function evolution due to pseudo-evolution (points) to the actual evolution of the mass function, as quantified by Tinker et al. (2008), shown by the lines. The bottom panels show the fractional difference between the points and solid lines for the corresponding redshifts (note that the scale is larger than in the residual panel of Fig. 6). The static halo model predicts an almost uniform evolution of the mass function across all masses which is significantly different from the actual evolution of halo masses, especially for the low-mass halos. This difference indicates that in addition to mass evolution, a substantial fraction of low-mass halos disappear due to merging with other halos. See §3.4 for a detailed discussion.



( $M \lesssim 2 \times 10^{12} h^{-1} M_\odot$ ) stays constant since  $z = 1$ . The common explanation for this observed non-evolution is that low-mass halos have already collapsed and do not physically grow in mass or number. However, it is clear from Fig. 3 that these halos do indeed undergo a significant mass evolution just due to pseudo-evolution. This conflict can be resolved by noting that, in the backward evolution case, the radii of the halos reduce as we evolve the masses to higher redshifts. This may uncover substructures in the outer part of the halos, which can potentially be counted as isolated halos at those higher redshifts. Thus, the non-evolution of the halo mass function at the low mass end must be a result of the fortuitous cancellation of the effect of mass evolution and the addition of low mass halos at the outskirts of bigger halos. Since we limited our analysis to use only isolated halos at  $z_0 = 0$ , we were not able to quantify this effect in the case of backward evolution.

However, we can investigate this effect by taking the static density profiles at  $z_0 = 1$  and evolving them forward to  $z = 0.5$  and 0 (*forward evolution*). In this case, the low-mass halos in the outskirts of larger mass halos should get absorbed. When we calculate the masses of halos by using static density profiles, we partially account for this effect by removing small mass halos whose centers end up within the radius of larger mass halos at  $z = 0.5$  and 0 as discussed in §3.2. The right hand panel of Fig. 7 shows the comparison between the forward evolution of the mass function of halos from the Bolshoi simulation, and the T08 mass function. The comparison shows that the discrepancy at the low mass end still persists, even after removing low-mass subhalos. The reason for this discrepancy is our implicit assumption that these density peaks are stationary. While we remove some subhalos as they are absorbed into larger halos, more halos which appear isolated at  $z = 1$  would suffer the same fate if we took their infall motion towards larger objects into account. This is consistent with our observation that high-mass halos (which subsume the lower mass halos in their outskirts) undergo some physical accretion in addition to the pseudo-evolution.

A naive comparison of the mass function differences between the forward and the backward evolution case at the low-mass end (see the bottom panels of Fig. 7) seems to suggest that the effect of removing subhalos is very small. However, we note that for the case of forward evolution, it is extremely important to remove such subhalos. Due to their proximity to a larger host halo, the outskirts of their density profiles contain significant mass contributions from the host halo. As the virial radius increases towards lower redshift, such subhalos gain a significant fraction of the host halo mass. If they are not removed, this unphysical mass evolution results in a large scatter in the mass evolution histories of the low mass halos. This scatter can have a large effect on the estimated mass function due to Eddington bias (see §3.2). Thus, *not* removing the 14% of halos which become subhalos by  $z = 0$  can lead to residuals of over 100% when comparing to the T08 mass function for the case of forward evolution.

#### 4. DISCUSSION

In the past two sections, we have demonstrated that pseudo-evolution due to changing reference density has a significant impact on the overall evolution of SO mass (often called *mass accretion history*). In this section, we expand on some of the implications of this result for our understanding of the scaling relations between various observables and halo mass. This includes the concentration-mass relation, the relation between stellar content and halo mass, and scaling relations for galaxy

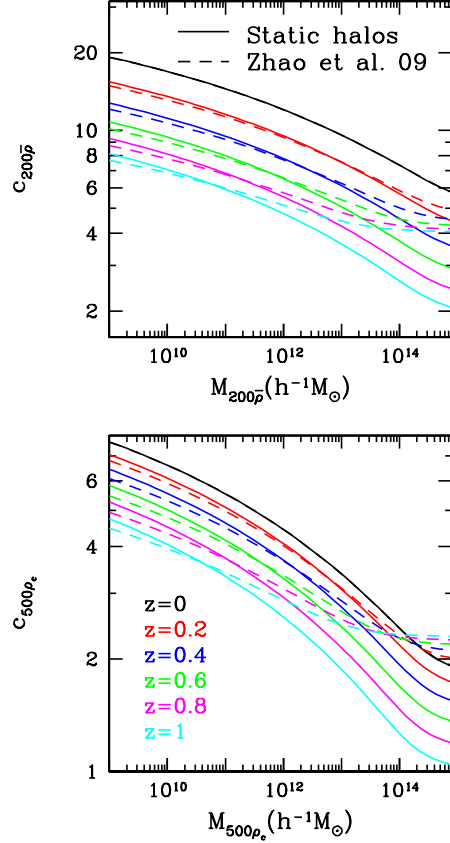


FIG. 8.— Evolution of the concentration-mass relation with redshift expected if the physical density around density peaks is unchanged over time. The solid lines show the analytical prediction of Equations (6)-(7). From top to bottom, the lines correspond to redshifts 0 to 1 in steps of 0.2. The dashed lines show the concentration-mass relation as a function of redshift from the physical model of Z09. This figure demonstrates that at low masses a large fraction of the observed evolution in the  $c-M$  relation is simply due to pseudo-evolution.

clusters.

##### 4.1. The concentration-mass relation

In the presence of pseudo-evolution, the virial radius of a halo grows with time, even though the halo's physical density profile (and thus its scale radius,  $r_s$ ) remain unchanged. However, since the virial radius *does* change due to pseudo-evolution, the concentration,  $c$ , grows at the same rate as the virial radius. Thus, we expect a significant evolution in the concentration-mass relation (hereafter  $c-M$  relation), even if the physical density profile of a halo remains unchanged.

We use the static halo model to estimate the magnitude of this evolution. The prediction of the static halo model for two mass definitions is shown with solid lines Fig. 8, and is in qualitative agreement with the evolution observed in numerical simulations such that the concentration of halos of a given mass decreases with increasing redshift. In reality, we expect that halos undergo some true physical evolution of mass due to physical accretion and merging, especially at the high-mass end, which will result in quantitative discrepancies between the *true* evolution of the  $c-M$  relation observed in numerical simulations and our static halo model predictions.

The dashed lines in Fig. 8 show the redshift-dependent  $c-M$  relation obtained from the models of Z09 which have been calibrated to reproduce the evolution of this relation in numerical simulations. The comparison clearly shows that most

of the evolution in the  $c - M$  relation at the low-mass end can be accounted for by pseudo-evolution of halo radius at different redshifts. For high-mass halos, however, the Z09 model captures the  $c - M$  evolution due to their significant physical mass accretion, and is thus not completely reproduced by our model of pseudo-evolution. Note that the static halo model quantitatively reproduces the evolution of the  $c - M$  relation for low-mass halos obtained by Bullock et al. (2001).

#### 4.2. Implications for galaxy formation

The total stellar content (or the stellar light) that we observe as galaxies in a halo is the integral result of the complex interplay between a variety of processes such as star formation, feedback from young stars, supernovae and supermassive black holes, and galactic outflows, all of which occur within dark matter halos. The redshift evolution of the scaling relation between this stellar mass (or luminosity) and halo mass can provide important observational clues regarding these different physical processes, in particular their efficiency as a function of halo mass. A number of studies have investigated the scaling relations between stellar mass and the mass of the halo they inhabit, and how these scaling relations change with time (Conroy et al. 2007; Brown et al. 2008; Behroozi et al. 2010; Moster et al. 2010; Abbas et al. 2010; Wake et al. 2011; Leauthaud et al. 2012; Yang et al. 2012b). However, when connecting the observed evolution of the scaling relations to the underlying physics, it is crucial to account for the pseudo-evolution of halo mass.

One of the striking implications of our results is that almost all of the mass evolution of galactic-sized halos ( $M_{200\bar{\rho}}[z = 0] \lesssim 10^{12} h^{-1} M_{\odot}$ ) since  $z = 1$  can be attributed to pseudo-evolution (see Fig. 3). The density profiles around the peaks of such halos have stayed static and not evolved physically (see also Prada et al. 2006; Diemand et al. 2007; Cuesta et al. 2008). Since physical accretion plays a minor role compared to pseudo-evolution, the impact of pseudo-evolution must be considered while interpreting the evolution of scaling relations and relating them to the underlying physical processes.

For example, the ratio of stellar mass to halo mass (SHMR), and its evolution with redshift, give a quantitative measure of how the star formation efficiency in a halo of given mass evolves with redshift. The peak of the star formation efficiency lies at roughly  $M_{200\bar{\rho}}[z = 0] \simeq 10^{12} h^{-1} M_{\odot}$ , and has been observed to shift to higher values from  $z = 0$  to 1 (Moster et al. 2010; Behroozi et al. 2010; Leauthaud et al. 2012). However, equal halo masses at two different redshifts correspond to different physical density peaks due to pseudo-evolution. Thus, the rate at which the similar density peaks become inefficient can differ from the estimates at fixed halo mass.

As a second example, let us consider the evolution of the SHMR at the more massive end. The stellar mass in such halos is dominated by satellite galaxies. Pseudo-evolution will lead to a constant fractional increase in both the stellar content and the halo mass if the distribution of satellite galaxies in and around halos, to first order, follows the matter density distribution, and if there is no radial segregation in the stellar mass of satellite galaxies. In this case, stellar and halo mass grow by the same factor, and the SHMR at the massive end undergoes an evolution with redshift which is qualitatively very similar to the evolution observed by Leauthaud et al. (2012). We will perform a quantitative comparison between the evolution of the SHMR due to pseudo-evolution of halo mass and

the observed SHMR evolution in future work.

#### 4.3. Implications for cluster scaling relations

As discussed in §1, scaling relations between the baryonic properties of clusters, such as X-ray temperature, gas mass, or entropy, and the mass of the cluster's dark matter halo, are key to our understanding of clusters and their use in cosmology. The simplest model for these scaling relations relies on the assumption that cluster halos collapse in a self-similar fashion (Kaiser 1986). In this model, the temperature  $T$ , for example, scales with halo mass as

$$T \propto \frac{M}{R} \quad (12)$$

where  $M$  denotes the mass within the radius  $R$ , and  $T$  is measured at  $R$  (KB12). If the halo mass is defined as a spherical overdensity mass (mass definitions such as  $M_{500\rho_c}$  or  $M_{2500\rho_c}$  are commonly used for clusters), the above scaling relation can be expressed as

$$T \propto (\Delta_c \rho_c)^{1/3} M_{\Delta}^{2/3}. \quad (13)$$

Noting that  $\Delta_c$  is a constant, but that the critical density evolves with  $E^2(z)$ , the evolution with redshift can be incorporated into the scaling relation as

$$T \propto [E(z) M_{\Delta}]^{2/3}. \quad (14)$$

Unfortunately, the  $E^2(z)$  factor only accounts for the evolution of  $R$  (due to the evolution of the  $\rho_c$  factor in Equation (13)), but *not* the pseudo-evolution of  $M_{\Delta}$ . Thus, for a halo whose density profile remains constant, the scaling relation predicts that the temperature will increase with time without any particular physical reason.

As shown in Fig. 4, the mass evolution of  $M_{500\rho_c}$  since  $z = 1$  is only partly due to pseudo-evolution, but also contains a large contribution from actual accretion. Nevertheless, pseudo-evolution accounts for a factor of almost two. If this contribution is not taken into account, pseudo-evolution will masquerade as a deviation from self-similar behavior. We intend to further investigate the impact pseudo-evolution has on scaling relations in the future. However, finding a mass definition which allows us to disentangle the effects of pseudo-evolution on cluster scaling relations is certainly a challenging task, especially because different cluster observables have different dependencies on the exact boundary used to define a halo.

## 5. CONCLUSION

Several authors have pointed out that spherical overdensity masses undergo an evolution due to the changing reference density (Diemand et al. 2007; Cuesta et al. 2008; Kravtsov & Borgani 2012). We have studied this spurious mass evolution quantitatively, and called it *pseudo-evolution*. Our main results are as follows.

1. We have demonstrated that a significant fraction of the halo mass growth since  $z = 1$  is due to pseudo-evolution rather than actual physical accretion of matter. For low-mass halos ( $\lesssim 10^{12} h^{-1} M_{\odot}$ ), the pseudo-evolution accounts for almost all of the evolution in mass since  $z = 1$ .
2. We found consistency between results derived from the simulated density profiles at  $z = 0$  and  $z = 1$ , indicating

that the physical shape of those profiles undergoes very little change over this epoch. We demonstrated that, at  $z = 1$ , the infall velocities around low-mass halos are insufficient to facilitate significant growth, which directly supports the observation of non-evolution of halo mass.

3. We have shown that a simple analytical model reproduces the pseudo-evolution observed in the Bolshoi simulation to a few percent accuracy.
4. We investigated the effect of pseudo-evolution on the halo mass function  $dN/d\ln(M)$ , and found it to simply shift the function toward higher masses. We propose that the non-evolution of the mass function at low masses since  $z = 1$  constitutes a fortuitous cancellation between pseudo-evolution and the absorption of small halos into larger halos.

We have left some promising avenues of research for future investigations. For example, in this paper we restricted ourselves to halos with spherical overdensity definitions. However, another popular way to identify and define halos is to use the friends-of-friends (FoF) algorithm, which relies on a linking length (which is fixed relative to the average inter-particle comoving separation) rather than an overdensity to define masses. It is well known that the density of FoF halos at their boundary depends upon linking length (see e.g. Frenk et al. 1988; Lukić et al. 2009; More et al. 2011a). Since the linking length parameter is constant in comoving coordi-

nates, its physical length increases with time as the scale factor ( $a = [1 + z]^{-1}$ ). This implies that for a static halo density profile, the extent of the FoF halo will increase with time, leading to pseudo-evolution. While explaining the mass evolution history of FoF halos, Fakhouri & Ma (2010) disentangle the growth into accretion of resolved halos and the accretion of a diffuse component. It is clear that the diffuse component will include a contribution from pseudo-evolution, thus overestimating the fraction of actually accreted diffuse matter compared to the fraction accreted from merging halos. Therefore, the pseudo-evolution of FoF halos will need to be carefully investigated.

The eventual goal of such investigations will be to find a mass definition which does not suffer from pseudo-evolution, which can be measured observationally and in simulations, and which allows the formulation of meaningful scaling relations. We hope to address this question in future work.

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## APPENDIX

## MASS EVOLUTION

The halo mass at a given redshift  $z_0$  is given by the integration of the density profile from the center to the radius of the halo  $R_\Delta$ , which depends upon the definition of the halo,

$$M_\Delta(z_0) = \int_0^{R_\Delta(z_0)} \rho(r', z) 4\pi r'^2 dr' . \quad (\text{A1})$$

The halo mass evolution from redshift  $z_0$  to a given redshift  $z$  can be decomposed into the following components

$$M_{\text{evo}}^{\text{tot}}(z) = M(z_0) + \int_{R_\Delta(z_0)}^{R'_\Delta(z)} \rho(r', z_0) 4\pi r'^2 dr' + \int_0^{R'_\Delta(z)} [\rho(r', z) - \rho(r', z_0)] 4\pi r'^2 dr' + \int_{R'_\Delta(z)}^{R_\Delta(z)} \rho(r', z) 4\pi r'^2 dr' \quad (\text{A2})$$

where  $R'_\Delta$  represents the estimate of the radius at redshift  $z$  calculated based upon the static halo model, which is slightly different from the *true* radius  $R_\Delta$  at redshift  $z$  due to physical evolution of the halo. The first integral represents the evolution of mass just due to the changing mass definition. The second integral accounts for the change in the halo mass within the radius  $R'_\Delta(z)$  due to change in density from redshift  $z_0$  to  $z$ . The third integral just adds up the mass between the halo boundary predicted by the static halo model,  $R'_\Delta$  and the true boundary,  $R_\Delta$ . If halos are entirely static as assumed in our analytical model, only the first integral contributes to the mass evolution and the rest of the integral terms are exactly zero. This prediction from the static halo model is shown as solid lines in Fig. 3, while the total mass evolution from models of Z09 is shown using blue dashed lines. The comparison shows that at low halo masses, the mass evolution is indeed dominated by the first integral term.